

# Quantum-Number Exotic Tetraquarks at Large $N_c$ in QCD(AS)

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It is shown that the large  $N_c$  limit of QCD with quarks in the two-index antisymmetric color representation [QCD(AS)] has narrow tetraquark states with exotic flavor quantum numbers. They decay into mesons with a width that is parametrically  $O(1/N_c^2)$ . Tetraquarks with non-exotic quantum numbers mix at leading order with mesons of the same overall quantum numbers. QCD(AS) at  $N_c = 3$  corresponds to ordinary QCD; its large  $N_c$  limit represents an alternative starting point for a  $1/N_c$  expansion to the standard one with quarks in the fundamental color representation.

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Our understanding of QCD has been greatly aided by the study of the large  $N_c$  (number of color charges) limit and the  $1/N_c$  expansion introduced by 't Hooft 40 years ago [1]. The limit takes  $N_c \rightarrow \infty$  and the coupling constant  $g \rightarrow 0$  in such a way as to keep  $g^2 N_c$  fixed. In a few special cases, such as QCD in 1+1 dimensions [2] or QCD in the limit of heavy quark masses [3, 4], the approach can be used as a basis for direct quantitative calculations of observables. However, typically the approach has been more useful in providing a qualitative understanding of many aspects of hadronic phenomena.

It has generally been thought that exotic hadrons are qualitatively understood in large  $N_c$  QCD, where by “exotic” one means hadrons that do not fit into a classification scheme based upon a simple quark model. Quantum-number exotic hadrons are ones that, by quantum numbers, *cannot* be  $\bar{q}q$  or  $qqq$  states. It has long been known that glueballs exist as long-lived particles (*i.e.*, resonances that are parametrically narrow) at large  $N_c$  and that, in this limit, they do not mix with mesons [3]. It has also been known since the late 1990s that quantum-number exotic “hybrid mesons”—mesons with quantum numbers that cannot be constructed out a pure  $\bar{q}q$  state in a simple quark model but require a “valence gluon”, such as  $J^{PC} = 1^{-+}$ —must exist as long-lived particles [5]. It has also long been believed that at large  $N_c$  tetraquarks—states composed of two quarks and two antiquarks—are forbidden at large  $N_c$  [3, 6].

While the commonly understood situation regarding glueballs and hybrids remains uncontroversial, recently Weinberg pointed out that the standard argument against the existence of resonant tetraquark states is not valid [7]. It is useful to summarize why tetraquarks were thought to be impossible at large  $N_c$ . Witten and Coleman [3, 6] both point out that, when a correlation func-

tion for a tetraquark source of the form  $J = \bar{q}q\bar{q}q$  is computed, the leading-order contribution is  $O(N_c^2)$  and consists of two disconnected quark loops, each one of which has the quantum numbers of an ordinary meson, and when cut, has a color-singlet  $\bar{q}q$  structure. From this argument, it was concluded that tetraquark sources produce only two-meson states and nothing else. However, as Weinberg observed, this conclusion does not follow: The leading *connected* contribution is  $O(N_c^1)$  and is not of a two-meson character; nothing in this argument excludes a tetraquark pole associated with it.

A nice way to see that Weinberg’s critique is correct is to consider the case in which the tetraquark source is a vector-isovector of the form  $J_a^\mu(\mathbf{x}) = \epsilon_{abc}(\bar{q}(\mathbf{x})\gamma_5\tau_b q(\mathbf{x}))\partial^\mu(\bar{q}(\mathbf{x})\gamma_5\tau_c q(\mathbf{x}))$ , which has the quantum numbers of the  $\rho$  meson. Its leading-order two-point correlation function is indeed represented as a disconnected  $O(N_c^2)$  diagram consistent with two pions in a vector-isovector configuration. However, one cannot conclude from this fact that there is no narrow vector-isovector hadron in the theory. Indeed, the  $\rho$  meson exists, couples to the source, and contributes to the correlation function at  $O(N_c^1)$ . Similarly, in the case of a quantum-number exotic tetraquark channel (*i.e.*, one whose quantum numbers cannot be obtained from a pure  $\bar{q}q$  state), one cannot conclude just based on the fact that the disconnected part of the correlator couples to two mesons that no tetraquark state exists.

Witten [3] gives a second argument that tetraquarks cannot exist as narrow resonances at large  $N_c$ : Meson-meson interactions are weak, with scattering amplitudes scaling as  $N_c^{-1}$ , and hence a two-meson interacting state does not have the strength to form a bound or resonant tetraquark. However, this argument is also spurious. Consider again two pions with vector-isovector quantum numbers. Despite the fact that the interaction is weak at large  $N_c$ , they do in fact resonate into a narrow  $\rho$  meson. In a similar manner, narrow [width  $O(N_c^{-1})$ ] quantum-number exotic tetraquarks coupled to two mesons with a strength  $O(N_c^{-1/2})$  are fully compatible with meson-

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meson interactions whose scattering amplitudes scale as  $N_c^{-1}$ .

For many observables, the large  $N_c$  world is known to behave similarly to the physical world of  $N_c = 3$ . Thus, one may be more likely to interpret some scalar mesons such as the  $f_0(980)$  as tetraquarks (as has been commonly suggested over the years [8–16]), if tetraquarks exist at large  $N_c$ . Weinberg’s analysis has sparked some interesting work on tetraquarks at large  $N_c$ . Two notable results are the observation by Knecht and Peris [17] that, if narrow tetraquarks do exist at large  $N_c$ , the parametric dependence of the width on  $N_c$  depends upon the flavor content of the state, and Lebed’s demonstration [18] that the existence of narrow tetraquarks requires a rather subtle  $N_c$  dependence of the coupling of paired bilinear sources to the tetraquark state in the limit in which the bilinear sources approach the same spatial point.

Weinberg’s analysis does not resolve a central question. It shows that previous attempts to rule out tetraquarks at large  $N_c$  are flawed, but it does not show that tetraquarks *do* exist. The purpose of the present note is to show that, while the status of tetraquarks in the most common extrapolation from  $N_c = 3$  to large  $N_c$  remains unresolved at present, there exists a different but equally valid extrapolation in which narrow tetraquarks can be shown necessarily to exist. The extrapolation in question puts the quarks into the two-index antisymmetric color representation [19–23] (rather than the color fundamental representation), and so is often denoted QCD(AS). At  $N_c = 3$  the two-index antisymmetric representation is three-dimensional, and the theory is identical to QCD. However, the extrapolation to large  $N_c$  is different and forms the basis for a distinct  $1/N_c$  expansion. A principal difference between the two expansions is that quark loops are not suppressed in QCD(AS), which leads to a different  $N_c$  counting for hadronic vertices involving mesons and to leading-order glueball-meson mixing. There has been considerable interest in QCD(AS) at large  $N_c$  due to its beautiful formal properties, including the emergence of various dualities [20–23]. At least for the case of baryons [24–26], it can be shown that mass relations based on QCD(AS) have considerable phenomenological predictive power [27, 28], as do relations based on the more standard variant. In general, one expects the expansion that does the better job describing the data for  $N_c = 3$  to depend upon the observable.

Since QCD(AS) includes quark loops at leading order, one expects that “ordinary” mesons and tetraquarks might be mixed. Thus, distinguishing between mesons and tetraquarks can be problematic. Here we focus on “true” tetraquarks—namely, ones that, at leading order in the  $1/N_c$  expansion, contain only components with at least two quarks and two antiquarks, and show that such states must exist as narrow hadrons at large  $N_c$  in QCD(AS). We accomplish this separation by considering states with so-called exotic quantum numbers—states that, by construction, cannot be composed of a single  $\bar{q}q$  pair, such as an isospin-two hadron.

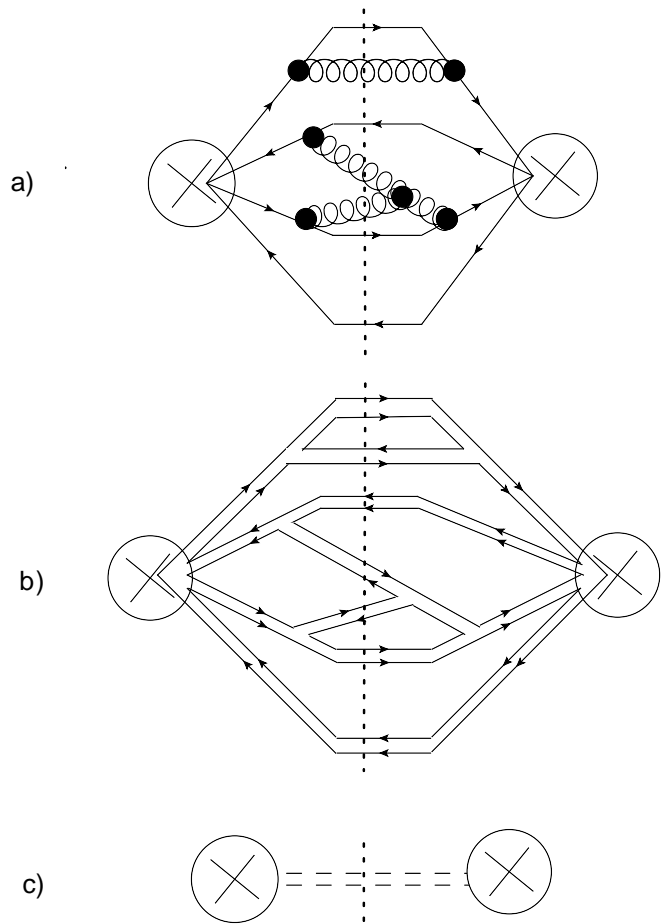


FIG. 1: Diagram (a) indicates a typical planar Feynman diagram that contributes to the leading-order (in  $N_c$ ) two-point correlation function for sources of the form of Eq. (1). The circle with a cross indicates the source. Diagram (b) shows the 't Hooft color-flow diagram associated with diagram (a). Diagram (c) is a hadronic-level depiction, indicating that the leading-order behavior is associated with the propagation of single hadron states. The vertical short-dashed lines indicate a cut of the diagram associated with one particular intermediate state.

The basic strategy is to use the same sort of diagrammatic analysis commonly used in standard large  $N_c$  studies of hadrons and to make appropriate modifications. One can repeat the standard double-line color-flow analysis of 't Hooft[1], but now quarks as well as gluons are represented by double lines. In contrast to the adjoint-representation gluons, for the quarks both color lines flow in the same direction. Consider the two-point correlation function for a tetraquark source operator of the following sort:

$$J(\mathbf{x}) = \sum_{A,B; a,b,c,d} C_{A,B} \bar{q}^{ab}(\mathbf{x}) \Gamma_A q_{bc}(\mathbf{x}) \bar{q}^{cd}(\mathbf{x}) \Gamma_B q_{da}(\mathbf{x}), \quad (1)$$

where lowercase letters represent fundamental color indices, the quark fields are antisymmetric in color ( $q_{ab} =$

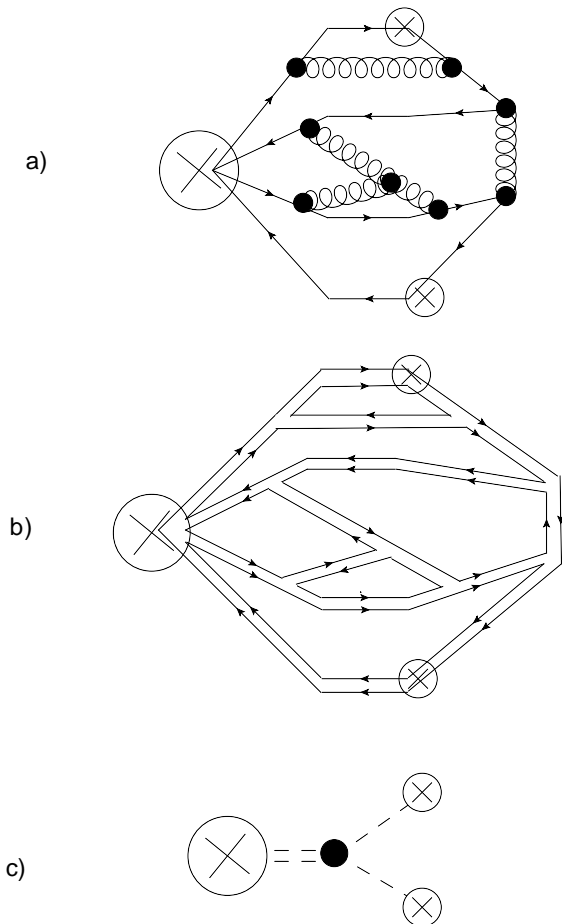


FIG. 2: Diagram (a) indicates a typical planar Feynman diagram that contributes to the leading-order (in  $N_c$ ) three-point correlation function for a tetraquark source of the form of Eq. (1) and two quark bilinear sources. The large circle with a cross indicates the tetraquark source, while the smaller ones indicate the bilinear sources. Diagram (b) shows the 't Hooft color-flow diagram associated with diagram (a). Diagram (c) is a hadronic-level depiction of the leading-order behavior.

$-q_{ba}$ ), explicit flavor and Dirac indices for the quarks are suppressed, and  $\Gamma_{A,B}$  represent matrices in Dirac and flavor space. Spin and flavor quantum numbers are fixed by the choice of  $C_{AB}$ . The key point is that the source  $J$  as a whole is a color singlet, but the colors couple the quarks in such a way that one cannot split  $J$  into two color singlets for  $N_c > 3$ . Note that color-fundamental quarks cannot be entangled in this way for any  $N_c$ , since Fierz re-ordering always allows such quarks with contracted color indices to be combined into color-singlet bilinears.

The two-point correlation function of the  $J$ 's is dominated at large  $N_c$  by planar diagrams connecting the two sources. As the sources involve four separate color indices that are summed over, one expects that these diagrams scale at leading order as  $N_c^4$ , which is indeed the case. As an example, consider diagram (a) in Fig. 1, which

contains 6 coupling constants. The color flow is shown in diagram (b), which has 7 color loops. The 7 color loops yield a factor of  $N_c^7$  while the 6 coupling constants contribute a factor of  $N_c^{-3}$ , yielding an overall scaling of  $N_c^4$ , as advertised. More generally, any diagram can be constructed by starting with a skeleton of no gluons and the minimum number of quark loops, and then adding in planar gluons or planar quark loops one at a time. Each of them adds a color loop (a factor of  $N_c$ ) and two coupling constants (a factor of  $N_c^{-1}$ ), and hence does not alter the  $N_c$  counting.

The central issue is the color structure of the states created by the source. Consider diagram (a) of Fig. 1 in more detail. The vertical line represents a possible cut that exposes the intermediate state created. Its color structure is illustrated in diagram (b); we see that the color structure of the quarks and gluons making up that state is  $\bar{q}^{ab} A_b^c q_{cd} A_c^d A_e^e A_f^f q_{ga}$ . The key point is that it is a single-color trace object. It cannot be split into two separate color-singlet combinations except due to sub-leading contributions in  $1/N_c$  in which two colors accidentally coincide. If one assumes confinement so that all quarks and gluons are bound into hadrons, this observation means that contributions from this cut of this diagram correspond to a single hadron. Moreover, it is easy to see that this result is generic: All cuts of all leading-order diagrams using the source  $J$  have a single-color trace structure. Thus, one concludes that the correlation function at leading order is saturated by single-hadron states; this result indicated by diagram (c) of Fig. 1. If the source creates states that include ones with exotic quantum numbers, one concludes that at large  $N_c$  quantum-number exotic tetraquarks must exist as narrow hadrons in the theory. This is the principal result of this note.

The parametric dependence upon  $N_c$  of the interaction of tetraquarks with themselves and with other hadrons can be determined by studying higher-point correlation functions. Note that from the analysis above,  $J$  creates a free [propagator  $\sim N_c^0$ ] tetraquark with an amplitude  $\sim N_c^2$  in QCD(AS), while standard meson and glue-ball sources create hadrons with an amplitude of  $\sim N_c^1$ . Consider, as a concrete example, the tetraquark-meson-meson vertex. One might think that a typical diagram contributing to three-point function can be obtained from a typical contribution to the tetraquark two-point function by simply removing a tetraquark source and adding two meson sources. However, this cannot be done: The tetraquark source  $J$  scrambles the colors of the various sources. To reconnect the colors when removing  $J$ , one needs to add at least one gluon exchange, as in going from the diagrams in Fig. 1 to those in Fig. 2. Note that, in adding the gluon, one does not change the number of color loops as compared to the two-point function [there are still 7 in diagram (b) of Fig. 2], but the graph has two additional coupling constants [there are 8 in diagram (a) of Fig. 2], which costs an additional factor of  $N_c^{-1}$ . Thus, the overall  $N_c$  scaling of the diagram is  $N_c^3$ . This

scaling is generic; the leading contribution to three-point functions with one tetraquark source and two ordinary meson sources is  $N_c^3$ .

At the hadronic level, this correlation function is dominated by a single tetraquark created by the source  $J$  with amplitude  $\sim N_c^2$  and each meson source producing a single meson with amplitude  $\sim N_c$ , for a total of  $N_c^4$ . The hadrons propagate and interact at a vertex, as in diagram (c) of Fig. 2. Together, the amplitudes for creating the hadrons ( $\sim N_c^4$ ) folded in with the propagation of each hadron ( $\sim N_c^0$ ) and the tetraquark-meson-meson interaction vertex must yield the full correlation function ( $\sim N_c^3$ ). Thus, the tetraquark-meson-meson interaction vertex must scale as  $1/N_c$ , and the decay width of a tetraquark into two mesons scales as  $1/N_c^2$ —which turns out to be the leading behavior for the tetraquark width: As noted earlier, at large  $N_c$  the tetraquark becomes stable. Using similar reasoning, it is easy to show that  $\Gamma_n$ , a general hadronic vertex with  $n$  hadrons (tetraquarks, glueballs, hybrids, and mesons) scales with  $N_c$  as

$$\Gamma_n \sim N_c^{2-n}. \quad (2)$$

In deriving Eq. (2), the key first step is to show that the  $N_c$  scaling of a diagram  $D$  containing  $n_T$  tetraquark sources and any number of meson, hybrid, and gluon sources scales as

$$D \sim N_c^{2+n_T}. \quad (3)$$

One consequence of Eq. (2) is that glueballs, hybrids and mesons all mix at leading order in QCD(AS), if allowed by quantum numbers (as can be seen from the two-point functions). Since the tetraquarks are exotic they do not mix with other hadrons. Note that tetraquarks sourced by a variant of Eq. (1) in which the bilinears are separately color singlets may still have exotic quantum numbers, but they would mix with conventional two-meson states at leading order, as discussed above.

The analysis goes through without substantial formal changes for the case of non-exotic quantum numbers, and Eq. (2) continues to hold. However, tetraquarks and mesons with non-exotic overall quantum numbers can mix at leading order, as indicated by Eq. (2). This is hardly surprising: Quark loops are not suppressed in QCD(AS). A  $\bar{q}q$  pair of the same flavor in a tetraquark can annihilate into a gluon, leaving behind a single  $\bar{q}q$  pair. In the case of non-exotic quantum numbers, such

a pair necessarily occurs. Thus, it is generally not possible to distinguish between tetraquarks and mesons with non-exotic quantum numbers.

One might hope that, if the theory has an exact flavor symmetry, there exist “true” tetraquarks with non-exotic overall quantum numbers that (at leading order) only contain components with two or more  $\bar{q}q$  pairs and thus do not mix at leading order with ordinary mesons. This scenario requires that the annihilation amplitude somehow cancels due to symmetry. However, such configurations do not appear to exist. Such a tetraquark would be natural for a source in which the color is in the configuration of Eq. (1), while the flavor is in a configuration such that neither  $\bar{q}q$  pair has a flavor-singlet component. At first sight, constructing such a state seems easy: For the case of two degenerate flavors, simply put each pair into an isovector configuration and then combine them to total isospin zero or one, yielding non-exotic overall quantum numbers. However, there are two distinct ways to form  $\bar{q}q$  pairs since each quark could pair with either antiquark; to prevent annihilation of a pair, the flavor configuration must be such that, with either pairing, no flavor singlet component exists for either pair. But no flavor configuration with this property exists.

This discussion suggests that the question of whether the  $f_0(980)$  or other mesons are tetraquarks is not entirely well posed at large  $N_c$  in QCD(AS); such states are mixed with both tetraquark and ordinary mesonic components, and the mixing is not parametrically suppressed. However, a scenario in which the mixing is *numerically* small and the state is dominated by the tetraquark configuration is fully consistent with what is known about QCD(AS) at large  $N_c$ . Finally, it should be noted that this type of analysis does not only imply tetraquarks. Hexaquarks, octaquarks and higher configurations with exotic quantum numbers exist as narrow states in QCD(AS) at large  $N_c$ , while such states with non-exotic quantum numbers mix with ordinary hadrons.

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